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LOW PROFILE,  
PRINTED CIRCUIT ANTENNAS

Liang C. Shen and Stuart A. Long

U.S. Army Research Office  
Grants DAHC04-75-G-0187  
and DAAG-29-75-0187

Final Report

October 1977

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Low Profile, Printed Circuit Antennas

Final Report

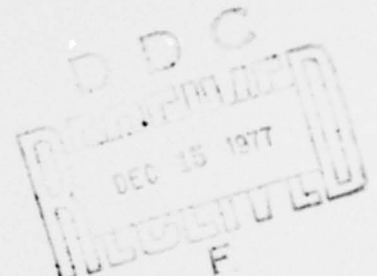
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## STATEMENT OF PROBLEM

The printed circuit or "microstrip" antenna has previously been shown to be a useful radiator for a wide variety of applications. Its utility lies mainly in its low profile and in its ease of construction, and ruggedness. Almost all investigations, however, have been superficial, in the sense that their only goal was for the particular design for yet another application. The task that remained was a careful investigation, both theoretical and experimental, of this class of antennas with the goal of explaining the basic nature of the radiation that was being observed empirically. Once the more basic features were unearthed the analysis would also be of aid in practical design problems.

## SUMMARY OF RESULTS

At an early stage in the project, it was decided that the proper course of action was to concentrate the effort on one particular shape of printed circuit radiator. To minimize the mathematical difficulties associated with finding an analytical solution a geometrically simple shape was needed. However, the shape chosen should also be of practical use. For these reasons a circular disc was chosen as the radiator to be investigated.

The theoretical investigation was begun with a zeroth order theory based on the modes of a circular disc cavity resonator. The lowest order mode was found to support fields which produced a broad far field pattern with a maximum normal to the plane of the disc. The electric and magnetic fields between the disc and the ground plane were found for this special lowest order resonant mode. Using these fields the currents on the radiator were calculated along with the far field radiation patterns. Once these were found all the radiation characteristics of the antenna were derived. Expressions for the total radiated power, directive gain, dielectric losses, conductor losses, efficiency, and Q-factor were then found as a function of the dielectric thickness, dielectric constant, and loss tangent as well as the frequency and size of the disc subject to the constraint that the resonance condition was maintained. Details and explicit analytical formulas are shown in Appendix I.

From earlier experimental work it was already obvious that the simple zeroth order approximation for the resonant frequency was not accurate enough. For this reason a first order solution was derived for the resonant frequency using a better approximation for the capacitance of the circular disc structure. An analytical solution was found which requires no numerical integrations or complicated computer calculations, and which is accurate enough to provide much more insight into the design problem. The derivation of this first order resonant frequency is shown in Appendix II.

The experimental investigation was carried out along two separate projects. The first was a model of sorts which consisted of a circular aluminum disc separated from a ground plane by a styrofoam slab. Several sizes of discs were used along with various thicknesses of the styrofoam. Each disc could be fed at several points along a radius of the disc and probes were constructed to measure the electric field between the disc and the ground plane. The driving point impedance, the resonant frequency, and the internal fields were all measured over a frequency range about the resonant point.

The second approach used actual antennas etched on printed circuit boards of various thicknesses. The far field radiation patterns were measured along with the impedance and resonant frequency. The results of both experiments were compared with the previously derived zeroth order far fields and the first order resonant frequency.



In each case reasonable correlation was found. The measured input impedance as a function of frequency was used to calculate an experimental value of the Q-factor. Its dependence on the size of the disc and the dielectric thickness was noted and compared with the previous theoretical calculation for the Q-factor. Again reasonable agreement was found. Details of both investigations are shown in Appendix III.

Although all the specific details and calculations were made for one particular shape, the results seem to apply to the entire class of printed circuit antennas. For example the behavior of the efficiency on the thickness of the dielectric substrate would be expected to be the same for a rectangular "patch" radiator. The same techniques could be applied in that case with a similar family of theoretical curves resulting.

Participating Scientific Personnel

Dr. Liang C. Shen, Principal Investigator

Dr. Stuart A. Long, Co-principal Investigator

Pierre B. Morel, Master of Science in Electrical Engineering

Martin R. Allerding, Master of Science in Electrical Engineering

Mark D. Walton, Master of Science in Electrical Engineering

Afroz, J. B. Zaman, Graduate Student, Degree in Progress

Frank Huang, Graduate Student, Degree in Progress

Faramarz Vaziri, Graduate Student, Degree in Progress

Brij Popli, Graduate Student, Degree in Progress

### Publications

L. C. Shen, S. A. Long, M. R. Allerding, and M. D. Walton, "Resonant Frequency of a Circular Disc, Printed-Circuit Antenna", Trans. Antennas Propagation, Vol. AP-25, July 1977, pp. 595-596.

Abstract - The resonant frequency is obtained in analytical form for a planar, circular disc antenna which is etched on a printed-circuit board so that the low-profile antenna is separated from the ground plane only by a thin layer of dielectric material. The formula is found to have an error of less than 2.5 percent when compared with experimental data.

### Manuscripts Submitted

Mark D. Walton, Stuart A. Long, and L. C. Shen, "An Experimental Measurement of the Radiation Fields and the Q-Factor of a Circular Disc Antenna".

Abstract - A radiating structure consisting of a circular disc over a ground plane separated by an electrically thin layer of dielectric has been constructed. Its radiation patterns have been measured and compared with previously derived theoretical far fields. In addition an experimental Q-factor was found from a measurement of the input impedance and also compared with theory.

Stuart A. Long, Liang C. Shen, and Pierre B. Morel, "The Theory of the Circular Disc, Printed Circuit Antenna".

Abstract - A planar, conducting structure an electrically small distance above a ground plane can be constructed to radiate in the direction normal to its plane while still retaining its low profile characteristics. A circular disc structure is analyzed theoretically to provide aid in the design of such antennas. The currents, fields, total radiated power, directive gain, losses, Q-factor and efficiency are all calculated at several frequencies for various values of the thickness, dielectric constant, and loss tangent of the material that separates the antenna from the ground plane.

## Presentations

Pierre B. Morel, Stuart A. Long, and Liang C. Shen, "A Theoretical Investigation of the Circular Disc Antenna," 1976 Region V IEEE Conference Digest, April 1976, pp. 286-289.

### Abstract

A circular, conducting disc above a ground plane is analyzed theoretically to provide a design procedure for practical antennas. The far fields, total radiated power, and power losses are found using the fields and currents within the cavity formed by the disc and the ground plane. Approximate results are also found for the case with the cavity filled with a dielectric material.

Mark D. Walton, Martin R. Allerding, Liang C. Shen, and Stuart A. Long, "An Experimental Investigation of the Circular Disc, Printed Circuit Antenna," 1976 Region V IEEE Conference Digest, April 1976, pp. 290-293.

### Abstract

Several circular disc, printed circuit antennas have been fabricated and investigated experimentally with regard to their driving point impedances and their far field radiation patterns. A model of such an antenna is also constructed to provide additional information concerning the dependence of the impedance on the position of the feed point.

Stuart A. Long and Liang C. Shen, "An Experimental and Theoretical Investigation of the Circular Disc, Printed Circuit Antenna," U. S. Army Workshop on Electrically Small Antennas Digest, May 1976, pp. 69-73.

### Abstract

A circular conducting disc over a ground plane is investigated as a low-profile antenna. The input impedance and radiation pattern are measured as a function of frequency and the thickness of the antenna. An approximate theoretical solution is also derived.

Stuart A. Long and Liang C. Shen, "The Circular Disc, Printed Circuit Antenna," IEEE AP-S International Symposium, Stanford Univ., June 1977.

### Abstract

Printed circuit antennas consisting of a planar radiating structure over a ground plane separated by an electrically thin layer of dielectric have been applied to a variety of systems. A particular configuration, the circular disc radiator, shown in Figure 1 was chosen to be analyzed theoretically with its radiation properties and resonant frequency calculated for dominant mode operation.

## Appendix I

### The Theory of the Circular Disc, Printed Circuit Antenna

Stuart A. Long, Ph.D.; Liang C. Shen, Ph.D.; Pierre B. Morel, M.S.

#### ABSTRACT

A planar, conducting structure an electrically small distance above a ground plane can be constructed to radiate in the direction normal to its plane while still retaining its low profile characteristics. A circular disc structure is analysed theoretically to provide aid in the design of such antennas. The currents, fields, total radiated power, directive gain, losses, Q-factor and efficiency are all calculated at several frequencies for various values of the thickness, dielectric constant, and loss tangent of the material that separates the antenna from the ground plane.

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# LIST OF SYMBOLS

$a$	radius of the circular disc
$d$	separation of disc from ground plane
$d_s$	skin depth of conductor
$\vec{E}$	total electric radiation field
$\vec{E}_a$	electric field at the aperture of the cavity
$E_o$	magnitude of electric field inside the cavity
$E_z$	axial electric field inside the cavity
$E_\theta$	electric radiation field in $\hat{\theta}$ -direction
$E_\phi$	electric radiation field in $\hat{\phi}$ -direction
$G_D$	directive gain
$\vec{H}$	total magnetic radiation field
$H_r$	radial magnetic field inside the cavity
$H_\phi$	circumferential magnetic field inside the cavity
$I_1$	first numerically calculated integral
$I_2$	second numerically calculated integral
$J_n$	Bessel function of order $n$
$J'_n$	derivative of the Bessel function with respect to its argument
$k$	wave number of the dielectric
$k_o$	wave number of free space
$K_r$	radial surface current on disc
$K_\phi$	circumferential surface current on disc
$\vec{M}$	equivalent magnetic surface current
$n$	mode number
$\hat{n}$	unit vector normal to the aperture
$P_D$	losses due to the imperfect dielectric
$P_L$	losses due to the imperfect conductors

$P_{\text{rad}}$	total radiated power
$Q$	total Q-factor
$Q_D$	Q due to dielectric losses
$Q_L$	Q due to conductor losses
$Q_{\text{rad}}$	Q due to radiation
$r$	cylindrical radial coordinate
$R$	spherical radial coordinate
$R_s$	surface resistivity of conductor
$W_T$	total stored electromagnetic energy in the cavity
$\delta$	loss tangent of dielectric ( $\tan \delta$ )
$\epsilon$	permittivity of the dielectric
$\epsilon_0$	permittivity of free space
$\epsilon'$	real part of dielectric permittivity
$\epsilon''$	imaginary part of dielectric permittivity
$\eta$	efficiency
$\theta$	spherical polar angle coordinate
$\mu$	permeability of the dielectric
$\mu_0$	permeability of free space
$\sigma$	conductivity of the conductor
$\phi$	cylindrical and spherical azimuthal angle coordinate
$\omega$	angular frequency

## 1. Introduction

Printed circuit antennas consisting of a planar radiating structure over a ground plane separated by an electrically thin layer of dielectric have been recently applied to a variety of systems. These antennas are low-profile, extremely rugged, and normally quite inexpensive to fabricate when standard printed circuit techniques, such as photo-etching, are used. They are capable of radiating quite efficiently in the direction normal to their plane without protruding any great distance. The major disadvantages are their narrow band characteristics and the associated difficulties in design resulting from this sensitive nature of the performance characteristics as a function of frequency.

One particular printed circuit antenna, the circular disc radiator, shown in figure 1 was chosen to be analysed theoretically with the ultimate goal being to provide a design procedure for practical antennas etched on printed circuit boards. Theoretical formulas are obtained for the associated currents, far fields, radiated power, directive gain, ohmic and dielectric losses, efficiency, and Q-factor. These parameters completely characterise the circular disc, printed circuit antenna.

## 2. Currents and Fields

The fields inside the region between a circular conducting disc and a ground plane have previously been found using an analysis of the resonance conditions of such a structure.<sup>1</sup> To retain the desirable low-profile characteristics the thickness of the antenna, which is the separation distance  $d$  of the disc from the ground plane, must remain electrically small. For this reason, field configurations between the plates having only circumferential and radial variations but no variation in the  $z$ -direction have been investi-

gated. The following fields are found:

$$E_z = E_0 J_n(kr) \cos n\phi \quad (1a)$$

$$H_r = \frac{-j\omega\epsilon n}{k^2 r} E_0 J_n(kr) \sin n\phi \quad (1b)$$

$$H_\phi = \frac{-j\omega\epsilon}{k} E_0 J'_n(kr) \cos n\phi \quad (1c)$$

where  $n$  is an integer,  $k = \omega\sqrt{\mu\epsilon}$ ,  $J_n$  is the Bessel function of order  $n$ , and  $J'_n$  is the derivative of the Bessel function with respect to its argument. The dielectric material is characterised by a permittivity  $\epsilon$  and a permeability  $\mu$  and a time dependence of  $e^{j\omega t}$  is assumed.

The surface currents on the disc can then be found with  $K_\phi = -H_r$  and  $K_r = H_\phi$ . The radial component of this current must vanish, however, at the edge of the disc which requires that:

$$K_r(r=a) = H_\phi(r=a) = 0; \quad J'_n(ka) = 0 \quad (2)$$

Thus for each modal configuration a particular radius can be found for resonance corresponding to zeros of the derivative of the Bessel function. The  $n=1$  mode has the lowest resonant frequency which corresponds to a minimum radius of  $ka = 1.84$ . This particular mode is usually excited in practical circular disc antennas. All formulas presented hereafter are for this dominant  $n=1$  mode. Thus, the frequency, dielectric constant, and radius of the antenna are always chosen in such a combination that  $ka = 1.84$ . The previously found surface currents on the disc can be calculated for this  $n=1$  mode and are shown in figure 2.

If one neglects the thin layer of dielectric outside the cavity formed by the disc and the ground plane, the radiation fields can be calculated.

The fields present at the aperture between the disc and the ground plane,  $\vec{E}_a$ , can be represented by an equivalent magnetic surface current  $\vec{M} = \hat{n} \times \vec{E}_a$  where  $\hat{n}$  is the outward pointing normal to the aperture in the  $\hat{r}$  direction. The integration of the equivalent magnetic surface current can be carried out analytically for the radiation fields.

$$E_{\theta} = \frac{-jE_0 e^{-jk_0 R} J_1(ka) \sin(k_0 d \cos \theta) a \cos \phi J_1'(k_0 a \sin \theta)}{R \cos \theta} \quad (3a)$$

$$E_{\phi} = \frac{jE_0 e^{-jk_0 R} J_1(ka) \sin(k_0 d \cos \theta) \sin \phi J_1(k_0 a \sin \theta)}{k_0 R \sin \theta} \quad (3b)$$

Details of the derivation are given elsewhere.<sup>2</sup> These fields are shown in the two principal planes for several values of dielectric constant in figures 3 and 4.

### 3. Total Radiated Power and Directive Gain

The total radiated power for the dominant mode can be found using Poynting's theorem and the previously calculated far fields. Only power radiated through the upper half sphere will be considered due to the presence of the ground plane.

$$P_{\text{rad}} = \frac{1}{2} \left\{ \frac{1}{2} \text{Re} \iint (\vec{E} \times \vec{H}^*) \cdot d\vec{s} \right\} \quad (4a)$$

$$P_{\text{rad}} = \frac{1}{4} \sqrt{\frac{\epsilon_0}{\mu_0}} \pi d^2 E_0^2 J_1^2(ka) [(k_0 a)^2 I_1 + I_2] \quad (4b)$$



where

$$I_1 = \int_0^{\pi} J_1^2(k_0 a \sin \theta) \sin \theta d\theta \quad (4c)$$

$$I_2 = \int_0^{\pi} \frac{\cos^2 \theta}{\sin \theta} J_1^2(k_0 a \sin \theta) d\theta \quad (4d)$$

The two expressions for  $I_1$  and  $I_2$  cannot be integrated analytically, but can be evaluated numerically.

The directive gain, defined as the ratio of the power density in the direction of maximum radiation to the average radiated power density, can be found from the previously found fields and the total radiated power.

$$G_D = \frac{\frac{1}{2} \operatorname{Re}(E_{\theta} H_{\phi}^* - E_{\phi} H_{\theta}^*)|_{\theta=0}}{P_{\text{rad}}/2\pi R^2} \quad (5a)$$

$$G_D = \frac{\frac{1}{480} d^2 E_0^2 (k_0 a)^2 J_1^2(ka)}{P_{\text{rad}}} = \frac{k_0^2 a^2}{k_0^2 a^2 I_1 + I_2} \quad (5b)$$

As long as  $ka$  remains equal to 1.84 and the antenna remains electrically thin the theoretical directive gain is not dependent on the exact value of the thickness. As seen in the patterns of figures 3 and 4 one would expect a lower value of gain for increasing dielectric constants. Its functional dependence is seen in figure 5 to vary from a maximum value of 4.84 for an air dielectric down to a value of approximately 1.7 for large dielectric constants. It should be noted that each change in dielectric constant requires a corresponding variation in the physical size of the disc so that

ka remains 1.84.

#### 4. Losses Due to Finite Conductivity and the Imperfect Dielectric

To obtain an approximation to the power losses due to the finite conductivity of the disc, the currents found for the ideal conductors are utilised. For a conductor with surface resistance  $R_s$  the losses in the disc and ground plane are:

$$P_L = 2 \frac{R_s}{2} \int_{r=0}^a \int_{\phi=0}^{2\pi} (K_\phi^2 + K_r^2) r d\phi dr \quad (6a)$$

The integral can be carried out analytically to yield the expression for the power loss.

$$P_L = \left( \frac{\pi f \mu_0}{\sigma} \right)^{1/2} \frac{E_0^2}{2} \frac{\epsilon_0}{\mu_0} \left\{ \frac{1}{2} J_1^2(ka) [(ka)^2 - 1] \right\} \quad (6b)$$

It is noted that these losses are independent of the thickness and that since  $ka = 1.84$  they are also independent of the dielectric constant but are proportional to  $f^{-3/2}$  and  $\sigma^{-1/2}$ .

For the case of an imperfect dielectric there exists the possibility of additional losses within the dielectric. These losses can be found by integrating the previously found fields over the volume of the cavity.

$$P_D = \frac{\omega \epsilon''}{2} \int_{vol} \vec{E} \cdot \vec{E}^* dv \quad (7a)$$

The dielectric material is characterised by  $\epsilon = \epsilon' - j\epsilon''$  or by the loss tangent,  $\tan \delta = \epsilon''/\epsilon'$ . Again, the integration is carried out analytically.

$$P_D = \frac{\tan \delta \, d \, E_o^2 \, J_1^2(ka) [(ka)^2 - 1]}{8 \mu_o f} \quad (7b)$$

For our imposed condition of  $ka = 1.84$  further simplification is possible.

$$P_D = 1.610 \times 10^{-4} \frac{d \, E_o^2 \tan \delta}{2f} \quad (7c)$$

It should be noted that the dielectric losses are not dependent on the actual value of the dielectric constant, but rather only on the ratio  $\epsilon''/\epsilon'$ , and that they are directly proportional to the thickness.

## 5. Efficiency

Allowing the possibility of power being lost in both the lossy dielectric and the imperfectly conducting walls the efficiency can be calculated.

$$\eta = \frac{P_{rad}}{P_{rad} + P_L + P_D} \quad (8)$$

This efficiency will depend on the thickness of the antenna, the frequency of operation, the permittivity and loss tangent of the dielectric material, and the conductivity of the radiating element. Any number of curves may be drawn to show these variations. Two such families of curves are shown in figures 6 and 7. In each case three of the five parameters are held constant. The values of  $d = .152$  cm,  $\epsilon_r = 2.55$ , and  $\tan \delta = 2 \times 10^{-3}$  correspond to typical values for microwave printed circuit board. The value of  $\sigma$  was not taken to be the theoretical value  $5.8 \times 10^7$  mho/m for bulk copper. Experience has shown that the effective value of the conductivity will be reduced due to surface roughness at the dielectric interface. A value of  $\sigma = 1.0 \times 10^7$  mho/m was chosen to be representative of this reduction. The efficiency is

seen to increase for higher frequencies, greater thicknesses, and, of course, smaller loss tangents. It should be noted again that curves for different frequencies must represent different physical devices since  $ka$  is required to equal 1.84.

## 6. Quality Factor $Q$ of the Cavity

The stored energy in the "cavity" between the radiating disc and the ground plane can be calculated and then compared with various energy losses to compute the values of several different  $Q$ -factors. The total stored electromagnetic energy,  $W_T$ , is independent of time. Therefore, it may be calculated from either the maximum electric fields or the maximum magnetic fields. Either can be evaluated analytically to give the following result:

$$W_T = \frac{E_o^2 d \pi}{4 \omega^2 \mu} [(ka)^2 - 1] J_1^2(ka) \quad (9)$$

$Q$ -factors may then be defined and calculated as follows with a remarkable simplification of the results.

$$Q_D = \frac{\omega W_T}{P_D} = \frac{1}{\tan \delta} ; \quad Q_{rad} = \frac{\omega W_T}{P_{rad}} \quad (10)$$

$$Q_L = \frac{\omega W_T}{P_L} = \frac{d}{d_s} ; \quad d_s = (\pi f \mu \sigma)^{-1/2} \quad (11)$$

A total  $Q$  can also be found which includes all the losses.

$$Q = \frac{1}{\frac{1}{Q_D} + \frac{1}{Q_L} + \frac{1}{Q_{rad}}} = \frac{\omega W_T}{P_D + P_L + P_{rad}} \quad (12)$$

Values of  $Q$  are shown graphically in figures 6 and 7 along with the efficiencies.

In figure 6 the total  $Q$  is dominated by the conductor losses for the smaller values of the thickness and by the radiation losses for the larger values. This results in a maximum value for  $Q$  in the middle range of thicknesses. The exact position of these maxima is dependent on the frequency. It should be noted that for all cases in figure 6,  $Q_D$  remains constant at a value of 500 and does not affect the value of  $Q$  to any extent. If the dielectric losses are allowed to increase, a sharp decrease in the value of  $Q$  is seen in figure 7. For the smaller values of the loss tangent  $Q_{rad}$  and  $Q_L$  again dominate resulting in a constant value of  $Q$  for each frequency.

#### 7. Acknowledgements

This work was supported in part by the U. S. Army Research Office under grants DNHCO4-75-G-0187 and DAAG-29-75-0187.

#### 8. References

- 1 WATKINS, J.: 'Circular Resonant Structures in Microstrip', Electron. Lett., 1969, 5, pp. 524-525.
- 2 MOREL, P. B.: 'A Theoretical Investigation of the Circular Disc Antenna', Masters Thesis, University of Houston, 1976.



## List of Legends

- Figure 1      Circular Disc Structure
- Figure 2      Surface Currents for  $n=1$  Mode
- Figure 3      Radiation Patterns of Circular Disc Antenna ( $E_{\theta}$ )
- Figure 4      Radiation Patterns for Circular Disc Antenna ( $E_{\phi}$ )
- Figure 5      Directive Gain as a Function of Dielectric Constant
- Figure 6      Efficiency and Q-Factor as a Function of Thickness
- Figure 7      Efficiency and Q-Factor as a Function of Loss Tangent

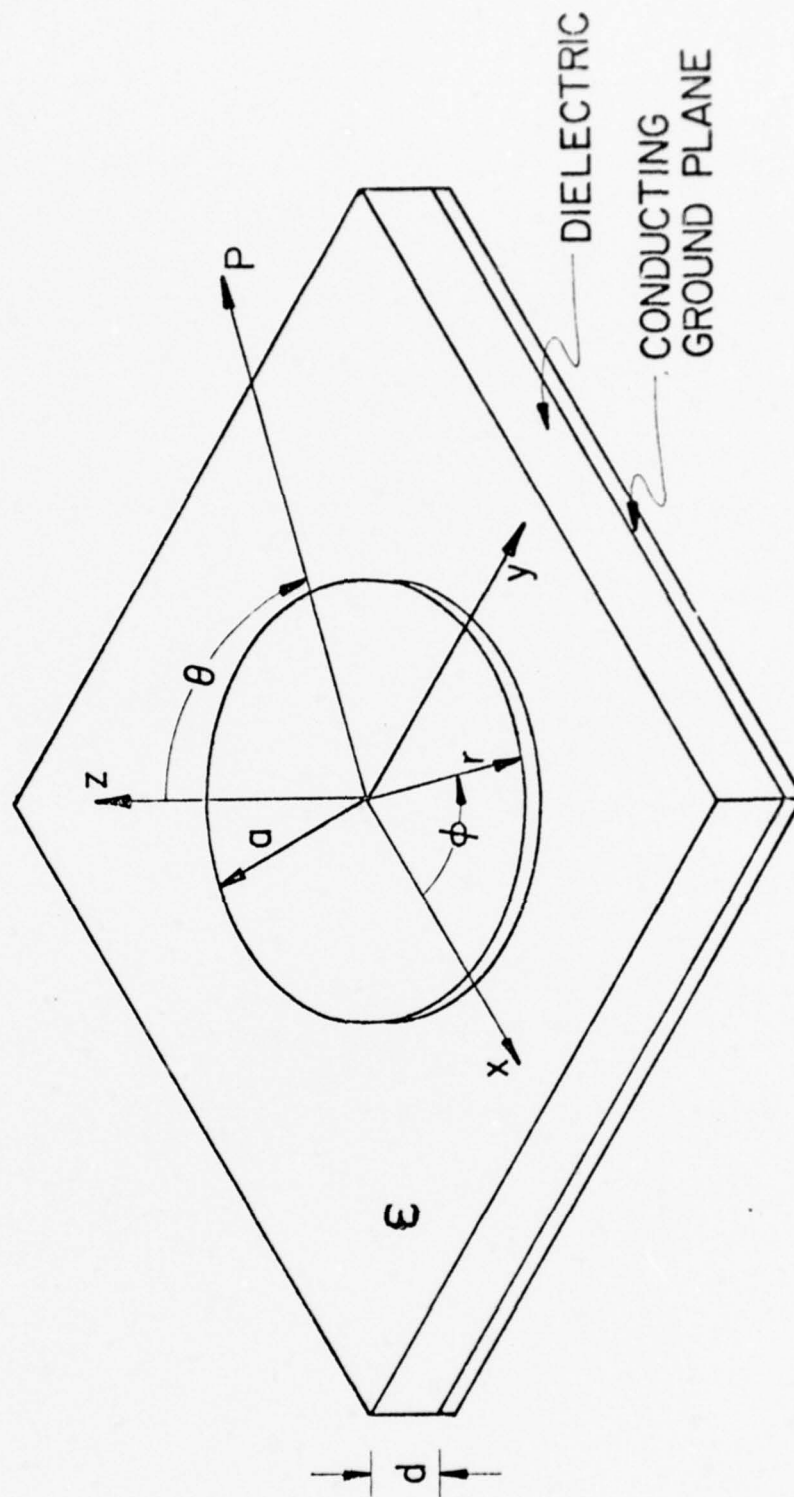


FIG.1 CIRCULAR DISC STRUCTURE

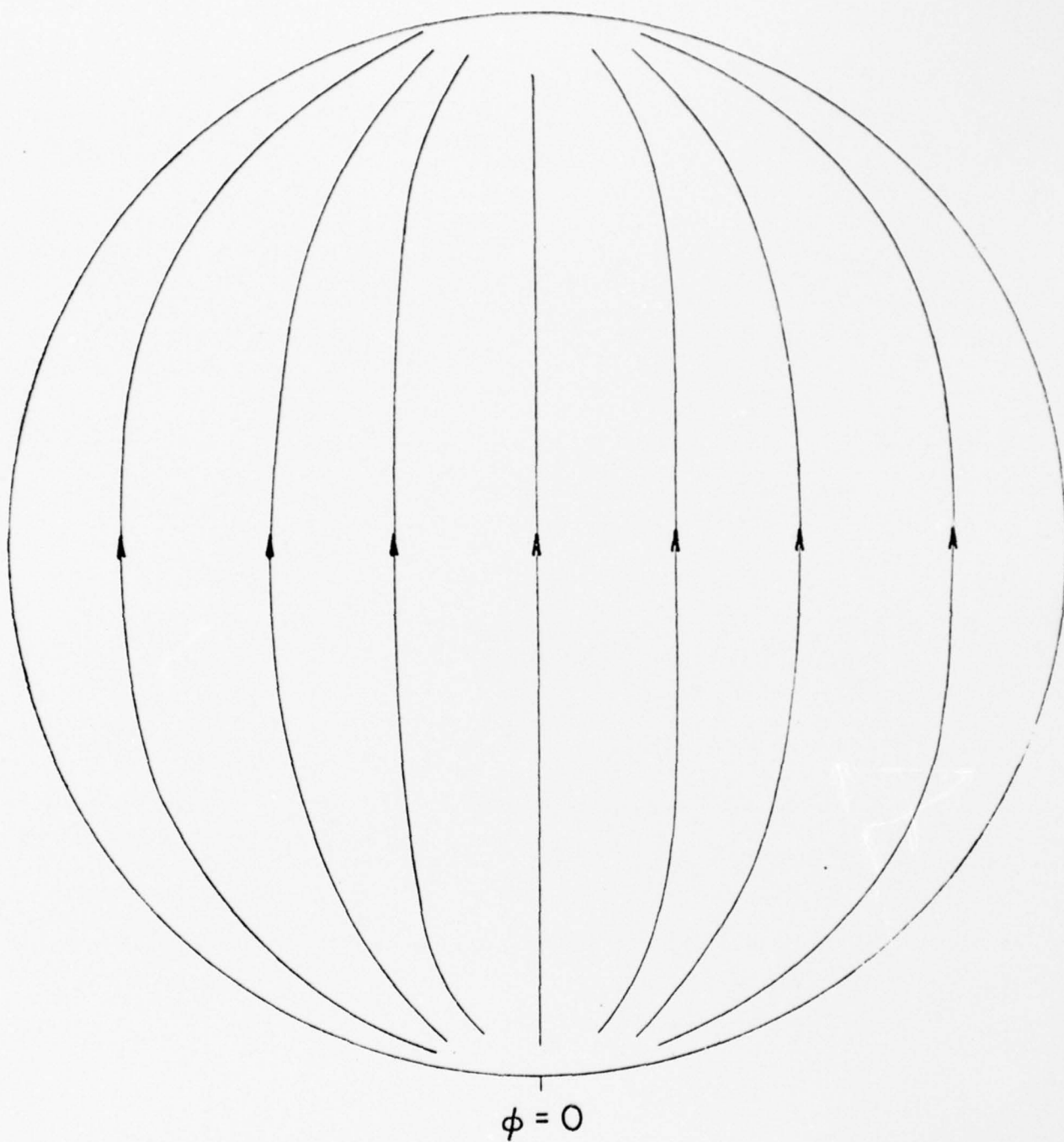


FIG. 2 SURFACE CURRENTS FOR N=1 MODE

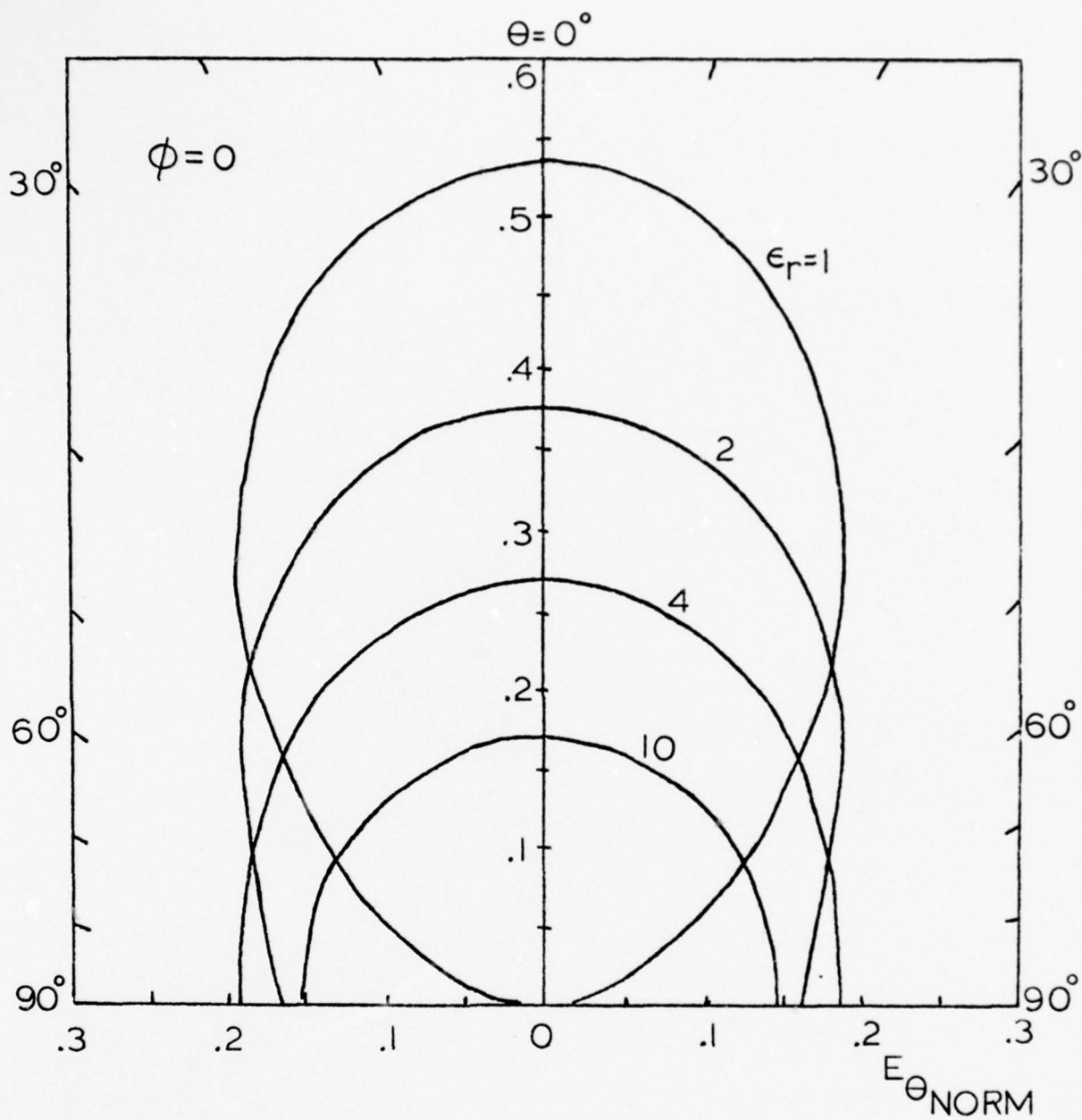


FIG 3

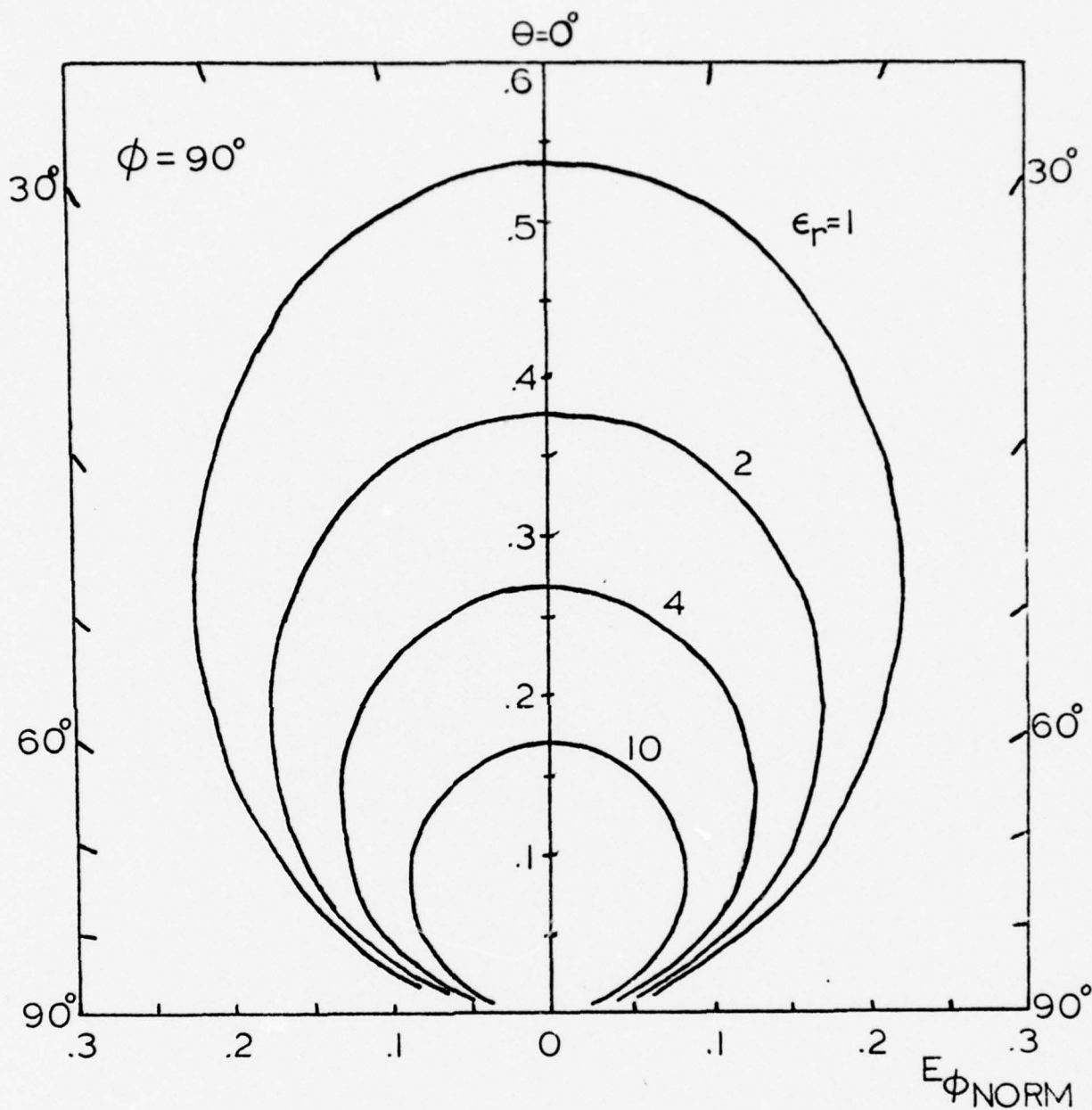


FIG 4



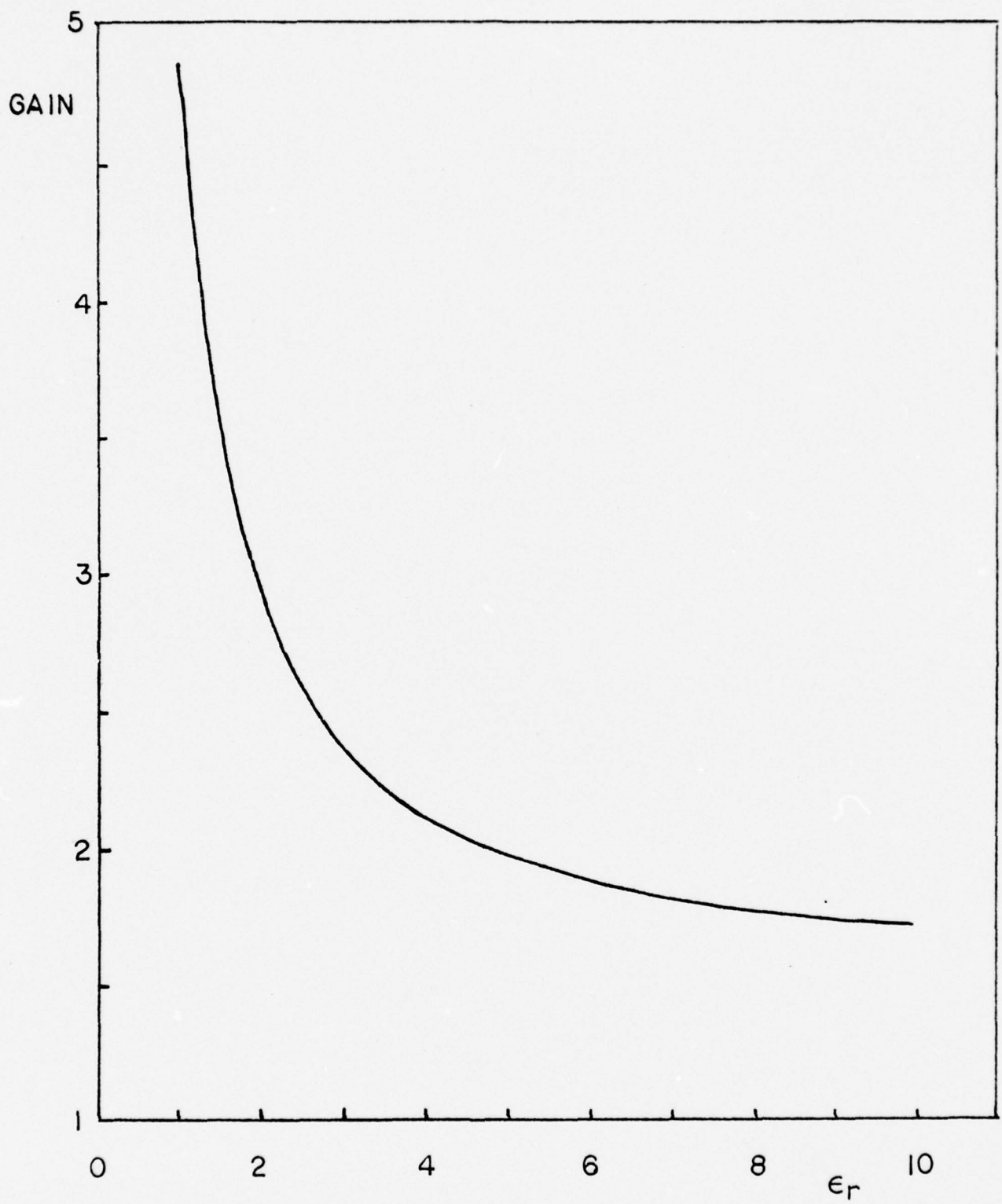


FIG5

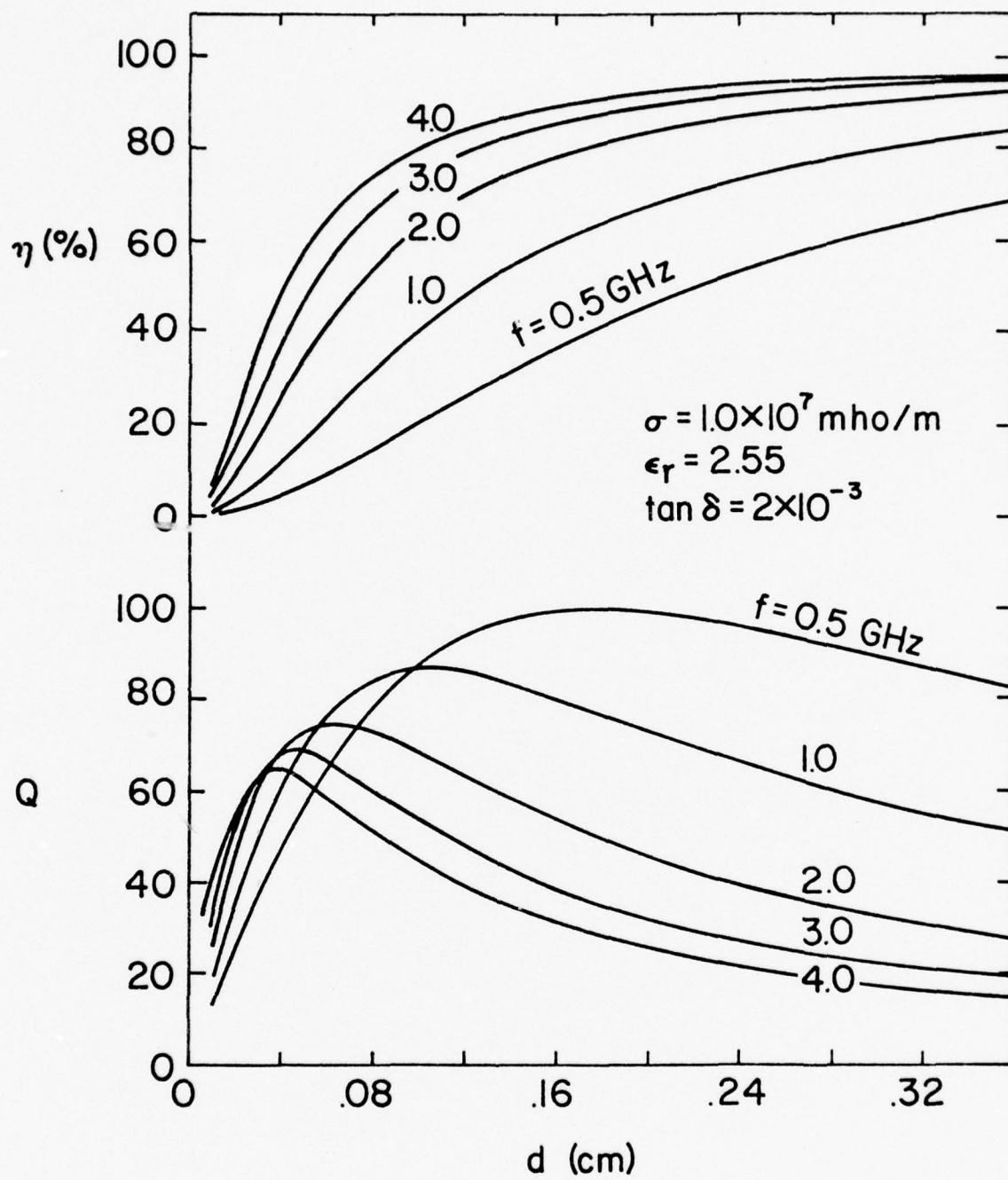


FIG 6

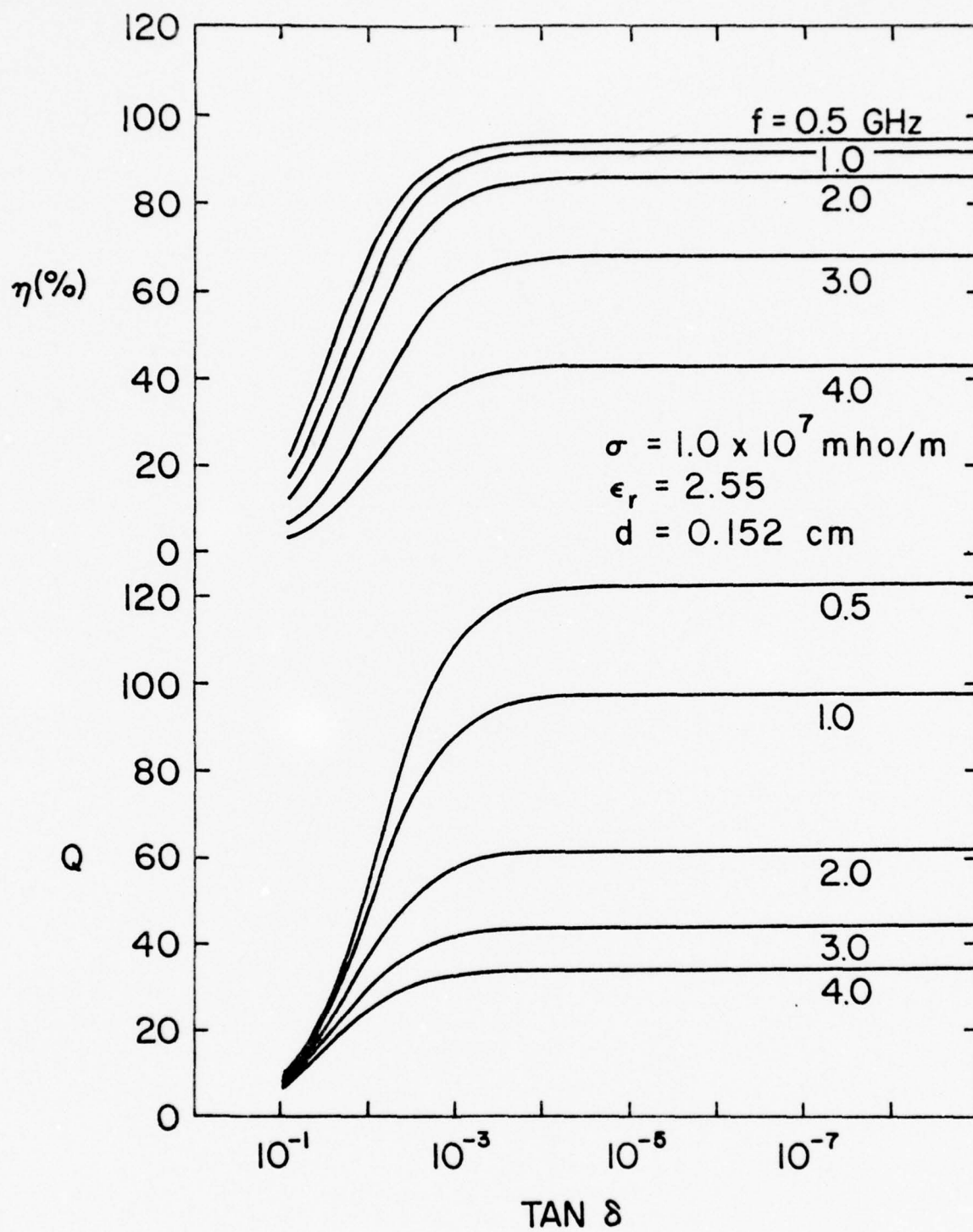


FIG 7

## INTRODUCTION

Planar, circular disc antennas which are etched on printed-circuit boards are being used in a variety of systems as low-profile antennas. Theoretical formulas have been obtained previously for the current distribution on the antenna, far-fields, radiated power, directive gain, and other characteristics [1]. The zeroth-order resonant frequency of the antenna is given by [2]

$$f^{(0)} = \frac{1.841}{2\pi a \sqrt{\mu\epsilon}} \quad (1)$$

where  $a$  is the radius of the disc as shown in Fig. 1, and  $\mu$  and  $\epsilon$  are the permeability and permittivity of the dielectric substrate of the printed-circuit board. Experiments have shown that the resonant frequency given by (1) is always higher than the measured data. The deviation of the zeroth-order resonant frequency from the measured data depends on the dielectric constant  $\epsilon$  and the separation  $d$ , as defined in Fig. 1. This error ranges from 4 percent for small  $d$  and high  $\epsilon$  to 17 percent for larger  $d$  and lower  $\epsilon$ . It is noted that larger separations and lower permittivities may result in higher radiated powers [1] and thus for practical radiators one may find large differences between the actual resonant frequencies and the zeroth-order theory.

Several methods have been reported in the literature to calculate the resonant frequency more accurately for the circular disc antenna. A numerical method was used by Itoh and Mittra [3]. An equivalent radius, equivalent dielectric constant concept was developed by Wolff and Knoppik [4], but an analytical formula for the equivalent dielectric constant was not given. More recently the resonant frequency was obtained by Borkar and Yang [5] by solving a dual integral equation. Numerical integration was then used to carry out certain integrals.

In this paper an analytical formula is derived for the resonant frequency. The formula is simply expressed in algebraic form involving no numerical integration and yields a result very close to the measured data.

## CAPACITANCE AND INDUCTANCE

The zeroth-order capacitance of the circular disk over a ground plane is

$$C^{(0)} = \frac{\pi a^2 \epsilon}{d} \quad (2)$$

Since the zeroth-order resonant frequency is given by (1) and

$$f = \frac{1}{2\pi\sqrt{LC}} \quad (3)$$

the zeroth-order inductance is given by

$$L^{(0)} = \frac{\mu d}{\pi z_0^2} \quad (4)$$

where  $z_0 = 1.841$  corresponds to the first zero of  $J_1'(z)$ , the derivative of the Bessel function of order 1.

## Resonant Frequency of a Circular Disc, Printed-Circuit Antenna

L. C. SHEN, S. A. LONG, M. R. ALLERDING, AND M. D. WALTON

**Abstract**—The resonant frequency is obtained in analytical form for a planar, circular disc antenna which is etched on a printed-circuit board so that the low-profile antenna is separated from the ground plane only by a thin layer of dielectric material. The formula is found to have an error of less than 2.5 percent when compared with experimental data.

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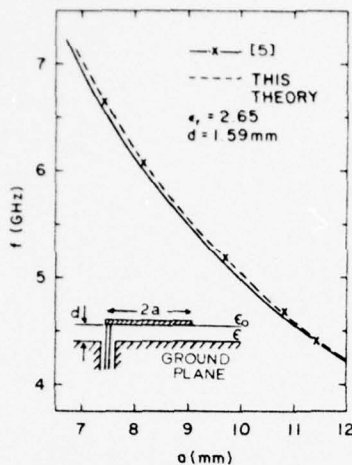


Fig. 1. Circular disc antenna and its resonant frequency as function of radius.

As shown in previous work [3]–[5], accuracy of the formula for the resonant frequency may be improved with a better approximation for the capacitance.

A simple algebraic formula for the first-order capacitance is available [4] when the dielectric substrate is replaced by air:

$$C^{(1)} = C^{(0)}(1 + \Delta) \quad (5a)$$

where

$$\Delta = \frac{2d}{\pi a} \left[ \ln \left( \frac{\pi a}{2d} \right) + 1.7726 \right] \quad \text{for } \epsilon = \epsilon_0. \quad (5b)$$

For  $\epsilon$  different than  $\epsilon_0$ , the capacitance is expressed by [5]

$$C = \frac{\pi a^2 \epsilon}{d} (1 - c_0 + c_0' - c_0'' + \dots) \quad (6)$$

where

$$c_0 = 2 \int_0^\infty \left[ \frac{(\epsilon/\epsilon_0) \tanh(pd/a)}{(pd/a)[\tanh(pd/a) + (\epsilon/\epsilon_0)]} - 1 \right] \cdot \frac{[J_1(p)]^2}{p} dp \quad (7)$$

and  $c_0'$  and  $c_0''$  are infinite series given in [5].

For small  $d/a$ ,  $c_0'$  and  $c_0''$  can be neglected and

$$c_0 = \frac{-2(d/a)}{(\epsilon/\epsilon_0)} \quad (\text{logarithmic term}).$$

Comparing the above result with (5b) suggests the following approximate formula for  $c_0$ :

$$c_0 = -\Delta = \frac{-2d}{\pi(\epsilon/\epsilon_0)a} \left[ \ln \left( \frac{\pi a}{2d} \right) + 1.7726 \right]. \quad (8)$$

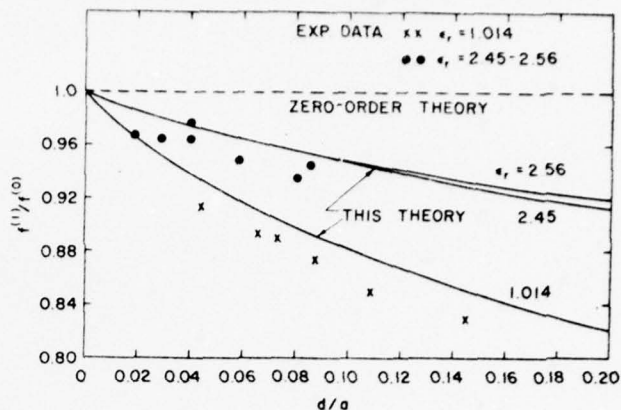


Fig. 2. Zeroth- and first-order resonant frequencies compared with experimental results.

The first-order resonant frequency  $f^{(1)}$  is then given by

$$f^{(1)} = \frac{1}{2\pi\sqrt{L^{(0)}C^{(0)}(1 + \Delta)}} \quad (9)$$

where  $\Delta$  is given by (8).

#### COMPARISON OF THEORY AND EXPERIMENT

The accuracy of the formula (9) is demonstrated in Figs. 1 and 2. In Fig. 1, the theoretical and experimental data reported in [5] are compared with the present theory with very good agreement resulting. In Fig. 2, experimental data obtained by Allerding [6] and Walton [7] are compared with the present theory. The experimental resonant frequencies were obtained by measuring the input impedances of circular disc antennas as the operating frequency was varied. All antennas were driven at the edge, as shown in Fig. 1. An antenna was considered to be at resonance when its input reactance was zero (or equivalently the input resistance was a maximum).

It is seen in Fig. 2 that while the zeroth-order formula gives an error of approximately 3 to 17 percent, the present theory predicts the resonant frequency with less than a 2.5 percent error.

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## Appendix III

### An Experimental Measurement of the Radiation Fields and the Q-Factor of a Circular Disc Antenna

Mark D. Walton, Stuart A. Long, and L. C. Shen

#### Abstract

A radiating structure consisting of a circular disc over a ground plane separated by an electrically thin layer of dielectric has been constructed. Its radiation patterns have been measured and compared with previously derived theoretical far fields. In addition an experimental Q-factor was found from a measurement of the input impedance and also compared with theory.

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## 1. Introduction

Printed circuit antennas consisting of a planar radiating structure over a ground plane have been used in a variety of systems [1,2]. One particular printed circuit antenna, the circular disc, shown in figure 1 is investigated experimentally. The far field radiation patterns and Q-factor were both measured and compared with existing theory.

## 2. Theoretical Results

The fields inside the region between a circular conducting disc and a ground plane have previously been found using an analysis of the resonance conditions of such a structure [3]. To retain the desirable low-profile characteristics the thickness of the antenna, which is the separation distance  $d$  of the disc from the ground plane, must remain electrically small. For this reason, field configurations between the plates having only circumferential and radial variations but no variation in the  $z$ -direction have been investigated. Under these restrictions the field inside the cavity can be calculated.

$$E_z = E_0 J_n(kr) \cos n\phi \quad (1a)$$

$$H_r = \frac{-j\omega\epsilon n}{k^2 r} E_0 J_n(kr) \sin n\phi \quad (1b)$$

$$H_\phi = \frac{-j\omega\epsilon}{k} E_0 J'_n(kr) \cos n\phi \quad (1c)$$

where  $n$  is an integer,  $k = \omega\sqrt{\mu\epsilon}$ ,  $J_n$  is the Bessel function of order  $n$ , and  $J'_n$  is the derivative of the Bessel function with respect to its argument. The dielectric material is characterized by a permittivity  $\epsilon$  and a permeability  $\mu$  and a time dependence of  $e^{j\omega t}$  is assumed.

The surface currents on the disc can be found and the radial component required to vanish at the edge of the disc.

$$\left. K_r \right|_{r=a} = \left. H_\phi \right|_{r=a} = 0; J_1'(ka) = 0$$

Thus for each modal configuration a particular radius can be found for resonance corresponding to zeros of the derivative of the Bessel function. The  $n=1$  mode has the lowest resonant frequency which corresponds to a minimum radius of  $ka = 1.84$ . This particular mode is usually excited in practical circular disc antennas. All formulas presented hereafter are for this dominant  $n=1$  mode, and thus the radius is adjusted so that  $ka$  remains equal to 1.84.

Using these zeroth order fields inside the cavity, the radiation fields, total radiated power ( $P_{rad}$ ), directive gain ( $G_D$ ), losses due to the finite conductivity,  $\sigma$ , of the radiator ( $P_L$ ), losses due to the imperfect dielectric ( $P_D$ ), the total stored electromagnetic energy in the cavity ( $W_T$ ), the quality factors due to radiation losses, conductor losses and dielectric losses ( $Q_{rad}$ ,  $Q_L$ ,  $Q_D$ ), the total quality factor ( $Q$ ), and the efficiency ( $\eta$ ) can all be calculated. These formulas have been previously derived [4] and are listed here for reference.

$$E_\theta = \frac{-jE_0 e^{-jk_0 r} J_1(ka) \sin(k_0 d \cos \theta) a \cos \phi J_1'(k_0 a \sin \theta)}{r \cos \theta} \quad (2a)$$

$$E_\phi = \frac{jE_0 e^{-jk_0 r} J_1(ka) \sin(k_0 d \cos \theta) \sin \phi J_1'(k_0 a \sin \theta)}{k_0 r \sin \theta} \quad (2b)$$

$$P_{rad} = \frac{1}{4} \sqrt{\frac{\epsilon_0}{\mu_0}} \pi d^2 E_0^2 J_1^2(ka) [(k_0 a)^2 I_1 + I_2] \quad (3a)$$

$$I_1 = \int_0^\pi J_1'^2(k_0 a \sin \theta) \sin \theta d\theta \quad (3b)$$

$$I_2 = \int_0^\pi \frac{\cos^2 \theta}{\sin \theta} J_1^2(k_o \sin \theta) d\theta \quad (3c)$$

$$G_D = \frac{\frac{1}{480} d^2 E_o^2 (k_o a)^2 J_1^2(ka)}{P_{rad}} \quad (4)$$

$$P_L = \left( \frac{\pi f \mu_o}{\sigma} \right)^{1/2} \pi \frac{E_o^2}{k_o^2} \frac{\epsilon_o}{\mu_o} \left\{ \frac{1}{2} J_1^2(ka) [(ka)^2 - 1] \right\} \quad (5)$$

$$P_D = 1.610 \times 10^{-4} \frac{d E_o^2 \tan \delta}{f} ; \quad \tan \delta = \epsilon''/\epsilon' \quad (6)$$

$$W_T = \frac{E_o^2 d \pi}{4 \omega^2 \mu} [(ka)^2 - 1] J_1^2(ka) \quad (7)$$

$$Q_D = \frac{\omega W_T}{P_D} = \frac{1}{\tan \delta} ; \quad \tan \delta = \epsilon''/\epsilon' \quad (8a)$$

$$Q_L = \frac{\omega W_T}{P_L} = \frac{d}{d_s} ; \quad d_s = (\mu f \mu_o)^{1/2} \quad (8b)$$

$$Q_{rad} = \frac{\omega W_T}{P_{rad}} \quad (8c)$$

$$Q = \frac{\omega W_T}{P_D + P_L + P_{rad}} \quad (9)$$

$$\eta = \frac{P_{rad}}{P_{rad} + P_L + P_D} \quad (10)$$



With the one exception of the total radiated power all of the above formulas were obtained analytically and can be evaluated directly. The integrals in equations (3b) and (3c) must, however, be evaluated numerically.

### 3. Experimental Measurements of Radiation Field

The far field radiation patterns were measured for a circular disc antenna etched on microwave printed circuit board. The diameter of the disc was 3.76 cm and the printed circuit board was 0.16 cm thick with a teflon-fiberglass dielectric ( $\epsilon_r = 2.47$ ). Both polarizations  $E_\theta$  and  $E_\phi$  of the electric field were measured at the resonance frequency of 2.815 GHz. The antenna was fed at the edge of the disc at  $\phi = 0$  by a coaxial cable from behind the ground plane. Several patterns were taken for each polarization as a function of  $\theta$  for various values of  $\phi$ . Several of these are shown in figures 2 and 3 along with the theoretical fields from equation (2). Quite reasonable agreement is seen with the single exception of  $E_\theta$  in the plane of the antenna ( $\theta = \pi/2$ ). This deviation between theory and experiment is due to the finite size ground plane used in the experimental model. Similar agreement was found for various other thicknesses and diameters[5].

### 4. Measurement of the Input Impedance and Q-Factor

The input impedance of several circular disc, printed circuit antennas was measured as a function of frequency using a network analyzer. Each antenna was edge fed and designed to radiate in the  $n=1$  mode ( $ka = 1.84$ ). Identical size discs were etched on three different thicknesses of printed circuit board and three different size discs were constructed on the same thickness board. A least-square fit model in the form of a parallel R-L-C circuit was then found from the experimental impedance points near resonance (see appendix). This model in turn allowed the experimental Q-factor to be



calculated and then plotted as a function of the thickness  $d$  in figure 4. In addition the theoretical curves for  $Q$  from equation (9) are also shown for each of the resonant frequencies of the discs. One set of curves is shown which corresponds to the exact parameters of the antenna,  $\epsilon_r = 2.47$ ,  $\tan \delta = 2 \times 10^{-3}$ , and  $\sigma = 5.8 \times 10^7$  mho/m. A similar set of curves is also shown with all parameters remaining the same except for the conductivity reduced to  $\sigma = 1.0 \times 10^7$  mho/m. Some reduction in the effective conductivity should be expected due to the slight irregularities in the surface between the copper disc and the dielectric. With this change in conductivity the values of experimental  $Q$ 's are seen to correlate very well with the family of theoretical curves.

## 5. Efficiency

Using an effective conductivity of  $\sigma = 1.0 \times 10^7$  mho/m the efficiency of the antennas can be calculated from equation (10). These values are shown in figure 5 for several different frequencies as a function of the thickness of the antenna. No direct experimental measurements have been made for the efficiency but the previous comparisons of  $Q$ -factors indicate that these similarly calculated efficiencies should also be accurate.

## Appendix

### Least-Square Fit Model for Calculation of $Q$ -Factor

For frequencies near resonance the impedance of a circular disc, printed circuit antenna behaves much like an R-L-C parallel resonant circuit. The input admittance of the circuit is given by

$$Y = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

For a given set of experimental data ( $Y_n = G_n + jB_n$ ) taken at a frequency  $\omega_n$

the quantity  $S$  is to be minimized.

$$S = \frac{1}{N} \sum_{n=1}^N \left| Y(\omega_n) - Y_n \right|^2$$

$\partial S / \partial G =$

The three element values can then be found from setting  $\partial S / \partial C = \partial S / \partial L = 0$ .

The resulting equations can be shown to be:

$$G = \frac{1}{N} \sum_{n=1}^N G_n$$

$$C \sum_{n=1}^N \omega_n^2 - \frac{N}{L} - \sum_{n=1}^N B_n \omega_n = 0$$

$$N C - \frac{1}{L} \sum_{n=1}^N \frac{1}{\omega_n^2} - \sum_{n=1}^N \frac{B_n}{\omega_n} = 0$$

Solving the last two equations for  $C$  gives:

$$C = \frac{\sum_{n=1}^N \frac{B_n}{f_n} - \frac{1}{N} \left( \sum_{n=1}^N \frac{1}{f_n^2} \right) \left( \sum_{n=1}^N f_n B_n \right)}{2\pi \left[ N - \frac{1}{N} \left( \sum_{n=1}^N \frac{1}{f_n^2} \right) \left( \sum_{n=1}^N f_n^2 \right) \right]}$$

Then using the calculated values of  $G$  and  $C$  from the resonant circuit model and the experimental resonant frequency the  $Q$ -factor may be calculated.

$$Q = \frac{2\pi f_o C}{G}$$

Results of this least-square-fit model are shown in figure A1. The real and imaginary parts of the impedance are shown for the parallel resonant circuit along with the experimental data. Good agreement is seen for all parts with the possible exception of a slight shift in the imaginary part, but this shift does not affect the calculation of the  $Q$ -factor.

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- Fig. 4     Q-factor versus thickness for various frequencies
- Fig. 5     Efficiency versus thickness for various frequencies
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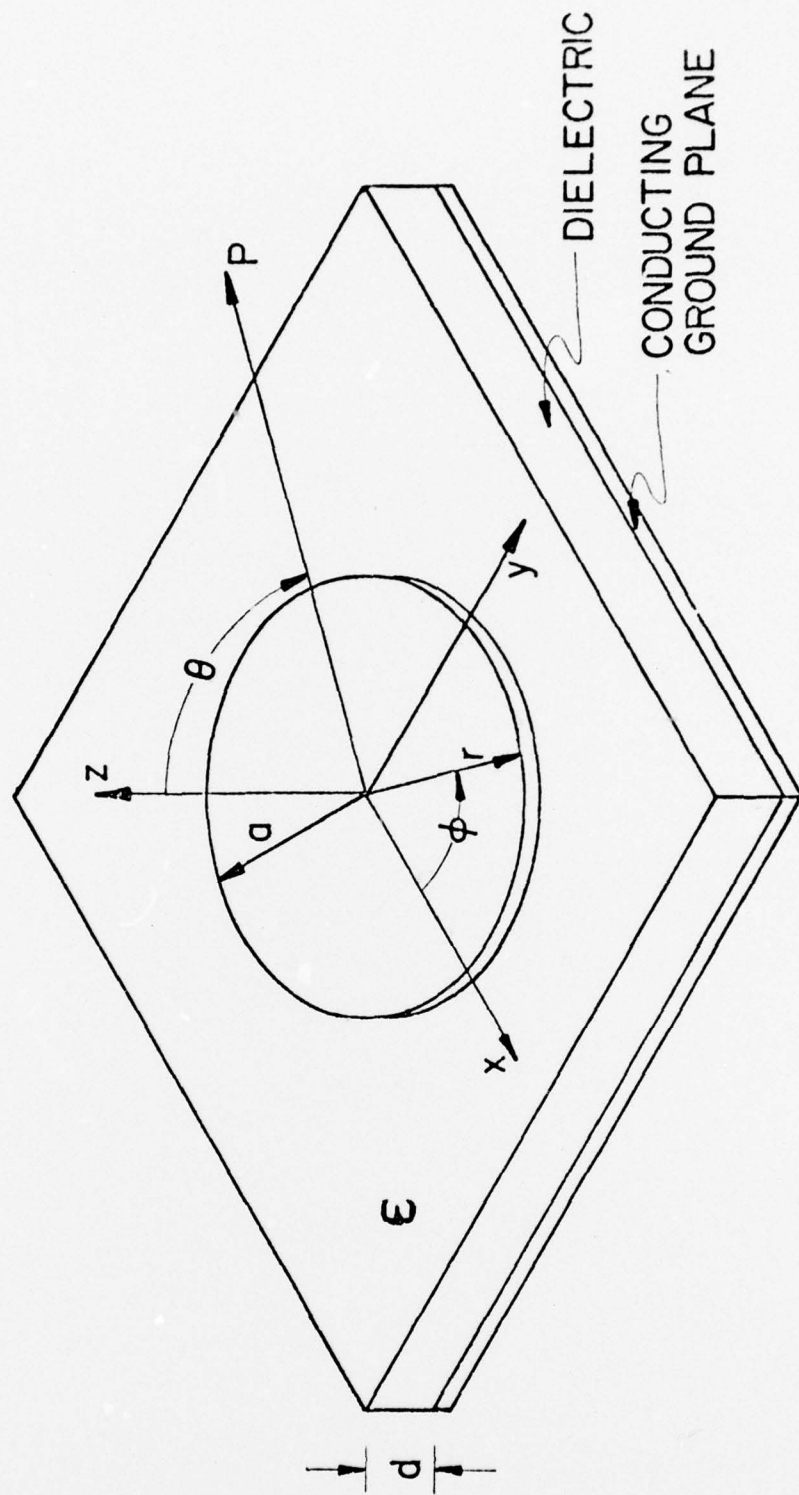
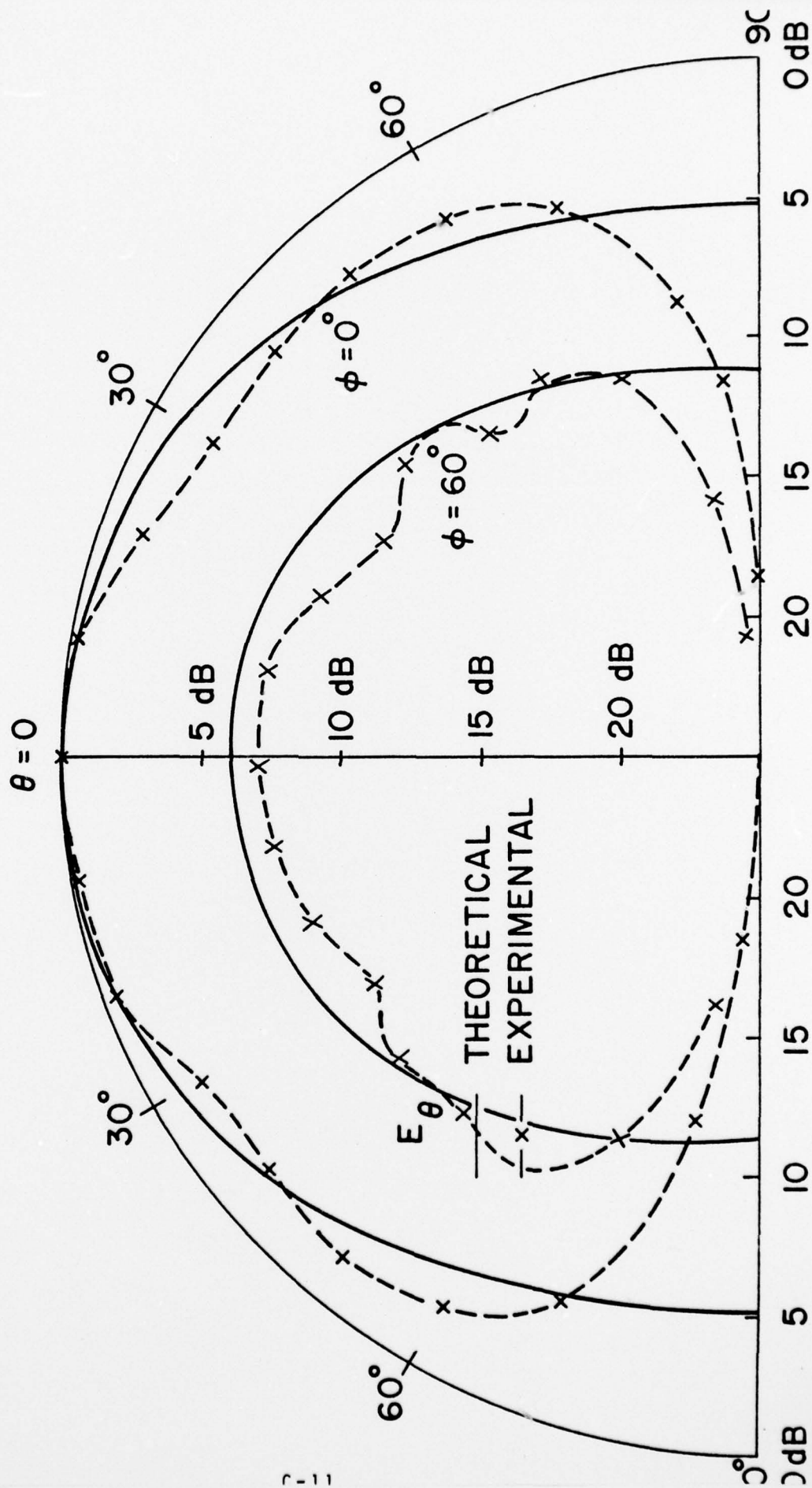


FIG.1 CIRCULAR DISC STRUCTURE





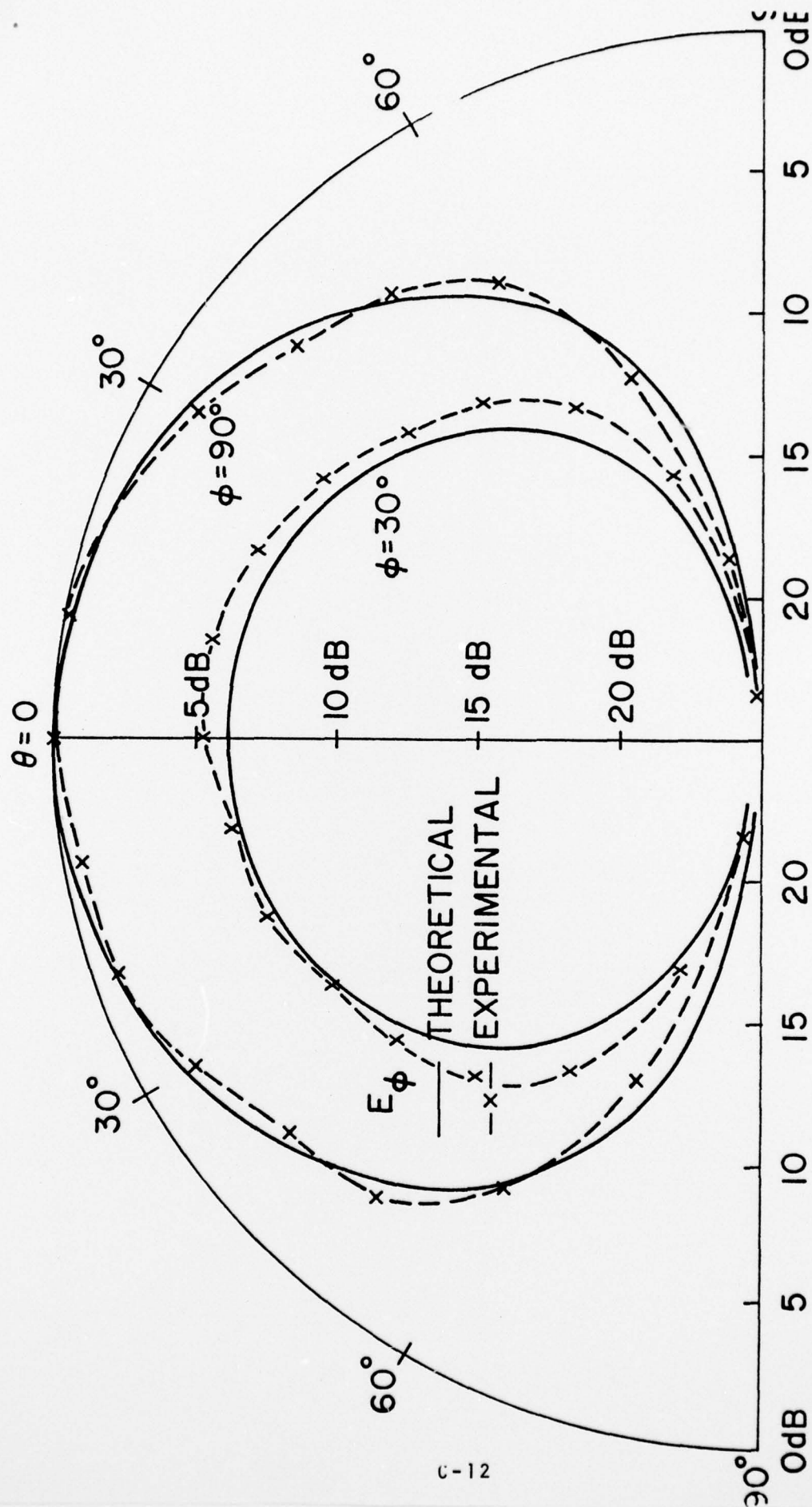


FIG. 3 FAR FIELD RADIATION PATTERNS

